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ACOUSTIC RADIATION AND SCATTERING FROM ELASTIC STRUCTURES.(U)
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ACOUSTIC RADIATION AND SCATTERING FROM ELASTIC STRUCTURES

BY

D.T. WILTON

ADMIRALTY UNDERWATER WEAPONS ESTABLISHMENT

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AMENDMENTS

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ERRATA

The following manuscript amendments are to be carried out:-

Page 5 - Para 15 line 3 - delete "$P_{scatt}(\underset{\sim}{r})$.", insert "$p_{scatt}(\underset{\sim}{r})$."

Page 7 - Para 20 equation (12) - delete "ds_r", insert "dS_r".

Para 25 equation (14) - delete "$\underset{\sim}{U}(\underset{\sim}{r},t) = e^{i\omega t}$ ...."

insert "$\underset{\sim}{U}(\underset{\sim}{r},t) \approx e^{i\omega t}$ ...."

Page 9 - Para 31 last line - insert "(26)" at right hand side of page.

Para 31 equation (26) - delete "$p = A^{-1} (\omega BX^{-1}L' q + p_{inc})$."

insert "$p = A^{-1} (i\omega BX^{-1}L' q + p_{inc})$."

Page 11 - Para 36 equation (35) - delete "$S = i\omega BX^{-1}L' \theta (A +$ ...."

insert "$S = i\omega BX^{-1}L' \theta (\Lambda +$ ...."

Para 36 equation (36) - delete "$(A + i\omega \theta^T (C + LA^{-1}BL')$ ...."

insert "$(\Lambda + i\omega \theta^T (C + LA^{-1}BX^{-1}L')$ ...."

Para 35 last line - delete "A is the ......"

insert "Λ is the ......"

Page 12 - Para 42 equation (42) - end of first line - delete "$v(2V-1)\, \underset{\sim}{r}_3$"

insert "$v(2v-1)\, \underset{\sim}{r}_3$"

Page 16 - Reference 12 line 2 - delete "User's", insert "Users'"

Page 17 - Reference 21 line 2 - delete "User's", insert "Users'".

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A.U.W.E. Tech. Note 574/78

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D. T. Wilton

CONTENTS

	Copy No.	
Duplicate Front Cover	(i)	
	(ii)	Blank
Title Page	(iii)	
Contents	(iv)-(v)	
	(vi)	Blank
Distribution (Detachable)	(vii)	
	(viii)	Blank
Précis	1	
Conclusions	1	
	2	Blank
Introduction	3	
Acoustic Equations	4-7	
Structural Equations	7-8	
Coupled Equations of Motion	8-11	
Numerical Implementation and Results	12-15	
Conclusions	15-16	
References	16-18	
Document Control Sheet	19	
	20	Blank
Abstract Cards (Detachable)	21	

ILLUSTRATIONS

Figure

1 (a) Surface acoustic finite elements;
(b) Structural finite elements.

2 Scattered surface pressure due to a plane wave incident upon a rigid sphere; wavenumber x radius = 2.0. Solid lines - analytical solution; crosses - computed solution.

3 Scattered surface pressure due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 2.0 cm, inner radius = 1.5 cm. Solid lines - analytical solution; crosses - computed solution.

4 Normal surface displacement due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 2.0 cm, inner radius = 1.5 cm. Solid lines - analytical solution; crosses - computed solution.

ILLUSTRATIONS (Cont'd.)

Figure	
5	Scattered surface pressure due to a plane wave incicent upon a rigid sphere; wavenumber x radius = 5.0. Solid lines - analytical solution; crosses - computed solution.
6	Scattered surface pressure due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 5.0 cm, inner radius = 4.921875 cm. Solid lines - analytical solution; crosses - computed solution.
7	Normal surface displacement due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 5.0 cm, inner radius = 4.921875 cm. Solid lines - analytical solution; crosses - computed solution.

ACOUSTIC RADIATION AND SCATTERING FROM ELASTIC STRUCTURES

PRÉCIS

1. This report presents a numerical technique for the linear dynamic analysis of a finite elastic structure immersed in an infinite homogeneous acoustic medium. It is required to determine the vibratory motion of the structure and also the associated acoustic field in the fluid, when the structure is either subjected to internal applied forces or is acting as a scatterer of an incident acoustic wave. A finite element analysis of the structure is matched at the structure-fluid interface with an integral equation representation of the exterior acoustic field, leading to a coupled system of equations which may be cast in either acoustic or structural form. The former approach is preferred here for which numerical results are presented when the method is applied to plane wave scattering by thick and thin elastic spherical shells.

CONCLUSIONS

2. By combining equations derived from a finite element analysis of a vibrating elastic structure with those from an integral equation representation of an infinite exterior acoustic field, an analysis of the coupled dynamic interaction problem has been shown to be feasible.

3. The structural and acoustic models may be chosen completely independently although the procedure does simplify if interpolation nodes are made to coincide, the coupling being defined through three interaction matrices X, L and L'. Thus advantage may be taken of existing computer program packages designed to solve the two uncoupled problems. In particular, it is hoped to combine the PAFEC[43] structural analysis program with the acoustic radiation/scattering program described in this report[38].

4. Some test examples for which analytic solutions are available have demonstrated the practicability and accuracy of the approach. It is hoped in the near future to be able to compare numerical and experimental results for some more complex structures in water. If an unacceptably large number of degrees of freedom for the structural model are required, it may then be necessary to consider the alternative modal approach.

INTRODUCTION

5. This report presents a numerical technique for the linear dynamic analysis of a finite elastic structure immersed in an infinite homogeneous acoustic medium. This problem commonly occurs in underwater acoustics where it is of considerable interest to determine the acoustic field both radiated by a submerged vibrating structure, and also scattered by a submerged elastic structure. It is well-known that the vibrational properties of a structure can significantly affect the scattered acoustic field, particularly when the acoustic medium is water, the impedance mismatch being much less than between a structure and air, and the assumption which is frequently made that the structure is perfectly rigid is often an oversimplification of the true situation.

6. Analytical approaches to such coupled structure-fluid interaction problems are almost invariably concerned with spherical or infinite cylindrical geometries for which the classical method of separation of variables is available[1], since in these cases normal structural modes, not coupled by radiation loading, exist. In particular, approximate shell theory has been extensively used to determine the response of thin elastic shells under acoustic loading.[2-4]

7. Until relatively recently little progress had been made towards the solution of interaction problems for other geometries. However, now that general numerical methods have been independently developed for both the structural and acoustic problems, the feasibility of combining these techniques to tackle the coupled problem has been realised.[5-12] For a complex structure subjected to known applied forces the finite element method[13] has become an accepted, well-proven, and highly successful analysis tool. Similarly, although certainly less widely applied, methods for determining the acoustic field radiated or scattered from a structure have been developed when boundary conditions on the structure surface are assumed to be known, e.g. rigid surface, soft 'pressure release' surface. These methods are usually based on integral equation formulations[14-17] and approximated using finite element type expansions.

8. Indeed, although the term 'finite elements' is only now being explicitly associated with methods for the solution of integral equations, if the name is understood to refer to the local piecewise nature of the approximation to the domain of the equation and also to the unknown function, and not necessarily associated with any particular method for determining the function parameters, such techniques have been in evidence for some time. Most commonly, when applying the finite element method to integral equations, the parameters are determined through collocation since the more familiar methods of Galerkin or using a variational principle require a further integration over the domain of the equation. When that domain is an arbitrary surface in three-dimensional Euclidean space over which the integrations are performed numerically, a repeated integration could be extremely costly in computer time. In fact Maxwell effectively solved the integral equation relating the potential of a thin charged conducting square plate to the surface charge density by a finite element approach in 1879[18]. Although the complete details are not given he apparently accounted for the expected singularity in the charge density at the edges and corners of the plate by using singular basis (shape) functions in these regions. Also product integration, which was introduced by Young[19] in 1954 as a technique for the solution of integral equations, is in its most popular form no more than a finite element method.

ACOUSTIC EQUATIONS

9. Small amplitude acoustic waves propagate through an ideal homogeneous fluid of density ρ_f and speed of sound c, according to the linear wave equation

$$\nabla^2 P(\underset{\sim}{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} P(\underset{\sim}{r},t) = 0 \tag{1}$$

where $P(\underset{\sim}{r},t)$ is the excess acoustic pressure at a position $\underset{\sim}{r}$ in the fluid at time t. Sound pressure is related to particle velocity $\underset{\sim}{V}(\underset{\sim}{r},t)$ through

$$\underset{\sim}{\nabla} P(\underset{\sim}{r},t) = -\rho_f \frac{\partial \underset{\sim}{V}}{\partial t}(\underset{\sim}{r},t). \tag{2}$$

10. For a single frequency harmonic time dependence of the form $e^{i\omega t}$, where ω is the angular frequency, equation (1) becomes the Helmholtz or reduced wave equation

$$(\nabla^2 + k^2)\, p(\underset{\sim}{r}) = 0 \tag{3}$$

where

$$P(\underset{\sim}{r},t) = p(\underset{\sim}{r})e^{i\omega t} \tag{4}$$

and $k = \omega/c$ is the acoustic wavenumber.

11. When the domain of the partial differential equation (3) is that infinite region exterior to the structure surface, techniques normally employed for the approximate solution of such equations (finite differences, finite elements etc) present obvious difficulties of implementation. A boundary must be introduced at some finite distance from the structure together with a boundary condition there which should approximate the true radiation condition at infinity, to ensure that all acoustic waves either radiated or scattered by the structure are outgoing there. Such an approach was suggested by Zienkiewicz and Newton[20] using a system of dashpots on the outer fluid boundary, although the correctness of this procedure has been questioned.[21] Even if the appropriate boundary condition could be derived[22], such methods necessarily require a large number of nodes, particularly in a three-dimensional situation, at each of which an approximation to the solution is produced. In practice however, it is often only the acoustic field at a selected number of nearfield positions and also the farfield radiation pattern that is of interest.

12. Thus the majority of workers in acoustics have chosen to reformulate the differential problem (i.e. differential equation plus sufficient boundary conditions) as an integral equation. This approach has the immediate advantage that the infinite exterior domain of the differential equation reduces to a finite domain of one dimension less (the structure surface) for the integral equation. Hence only surface values are initially calculated from which the field at any positions of interest may be evaluated through the integral representation inherent in the approach. Also, the exact radiation condition is automatically included in any integral equation formulation.

13. Unfortunately, all the classical formulations of the Helmholtz equation in an exterior region as an integral equation - equivalent single or double

layer source distributions or via Green's Theorem - break down either through non-uniqueness or non-existence of their solutions at certain wavenumbers. This failure is completely non-physical in nature, being solely due to the integral equation formulation, and is associated with the existence of eigen-solutions of the Helmholtz equation within the region interior to the surface of interest.[23,24] Various 'improved' formulations have been proposed which are either designed to mitigate the problem at low frequencies,[14,25] or to eliminate it completely.[26-28] These latter methods require considerably more computational effort to implement and have as yet not formed the basis of any practical acoustic radiation/scattering computer program, although test programs have been written and applied to some simple problems in two[29] and three[30] dimensions. A comprehensive feasibility study for such a program package has recently been completed.[31]

14. In order to illustrate the coupling procedure proposed here, the integral equation approach due to Schenck[14] is employed, although within the possible frequency limitations previously mentioned any available integral equation formulation could be used.

15. Consider an acoustic wave ($p_{inc}(\underset{\sim}{r})$) incident upon the closed surface S of the elastic structure, resulting in a scattered acoustic wave with pressure $p_{scatt}(\underset{\sim}{r})$. Then, from Green's Theorem, the total acoustic pressure $p(\underset{\sim}{r})$ ($= p_{inc}(\underset{\sim}{r}) + p_{scatt}(\underset{\sim}{r})$) satisfies the Helmholtz integral formulae[32]

$$\int_S \left\{p(\underset{\sim}{r})\frac{\partial G}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}') - \frac{\partial p}{\partial n_r}(\underset{\sim}{r})G(\underset{\sim}{r},\underset{\sim}{r}')\right\} dS_r = \begin{cases} p(\underset{\sim}{r}') - p_{inc}(\underset{\sim}{r}') & \underset{\sim}{r}' \text{ in E} \quad (5a) \\ \frac{\alpha}{4\pi} p(\underset{\sim}{r}') - p_{inc}(\underset{\sim}{r}') & \underset{\sim}{r}' \text{ on S} \quad (5b) \\ -\, p_{inc}(\underset{\sim}{r}') & \underset{\sim}{r}' \text{ in I} \quad (5c) \end{cases}$$

where E is the infinite acoustic medium exterior to S, I is the interior of the surface S and α is the solid angle subtended by the acoustic fluid at the point $\underset{\sim}{r}'$ on S (for a smooth surface, $\alpha = 2\pi$ everywhere).

$$G(\underset{\sim}{r},\underset{\sim}{r}') = \frac{e^{-ik|\underset{\sim}{r}-\underset{\sim}{r}'|}}{4\pi|\underset{\sim}{r}-\underset{\sim}{r}'|} \qquad (6)$$

is the three-dimensional free-space Green's function for the Helmholtz equation (3), and $\frac{\partial}{\partial n_r}$ denotes differentiation in the outward normal direction ($\underset{\sim}{n}$) at $\underset{\sim}{r}$ on S (i.e. from S into the fluid). In the case of pure radiation the total acoustic field $p(\underset{\sim}{r})$, which now consists of radiated pressure only, also satisfies the Helmholtz formulae (5) with the term $p_{inc}(\underset{\sim}{r})$ absent.

16. On any part of the surface S which has a well-defined normal direction, the relation (2) gives

$$\frac{\partial p}{\partial n_r}(\underset{\sim}{r}) = -i\omega\rho_f\, v(\underset{\sim}{r}) \qquad (7)$$

where $v(\underset{\sim}{r})$ is the outward normal surface velocity.

17. Hence, equations (5b) and (5c), known as the Surface Helmholtz Equation (SHE) and Interior Helmholtz Relation (IHR) respectively, relate the acoustic pressure on the structure surface to the normal surface velocity. Both have been used as the basis for the solution of acoustic radiation or scattering problems when either the surface pressure or velocity is known, although neither is entirely adequate.[33-36,16] The SHE is the most suitable for numerical treatment but exhibits non-uniqueness of the solution, whenever the wavenumber k is an eigenvalue of the Dirichlet eigenproblem for the interior region I[23]. The IHR can be shown to possess a unique solution if it is satisfied at all interior points[35] (for an axisymmetric problem it is only necessary to enforce the relation at all points along the axis of symmetry) but suffers from numerical instability due to its similarity to an integral equation of the first kind with a non-singular kernel.

18. The idea of Schenck was to combine the numerical stability of the SHE with the uniqueness property of the IHR by overdetermining the set of linear equations obtained from a numerical approximation of the SHE with a small number of additional equations derived from the IHR. The resulting linear system is then solved by a least squares procedure. This method is generally known by the acronym CHIEF (<u>C</u>ombined <u>H</u>elmholtz <u>I</u>ntegral <u>E</u>quation <u>F</u>ormulation). Schenck proved that at a critical wavenumber only the required solution of the SHE will simultaneously satisfy the IHR, provided the IHR is not evaluated on a nodal surface of the relevant interior eigenfunction.

19. In practice the CHIEF method is satisfactory only at relatively low frequencies when a sufficient number of correctly positioned interior nodes can be selected if required. Jones[25] has suggested a systematic approach to this choice.

20. In order to obtain a discrete matrix approximation to the SHE and IHR a finite element method is used. Approximations to the surface acoustic pressure and normal surface velocity are assumed in the forms,

$$p(\underset{\sim}{r}) \approx \sum_{i=1}^{m} p_i \, \phi_i(\underset{\sim}{r}) \tag{8}$$

$$v(\underset{\sim}{r}) \approx \sum_{i=1}^{m} v_i \, \chi_i(\underset{\sim}{r}) \tag{9}$$

where $\phi_i(\underset{\sim}{r})$ and $\chi_i(\underset{\sim}{r})$ are scalar basis functions defined on some surface S* approximating S. Collocation of the SHE (5b) at m surface points $\underset{\sim}{r}_i$, i=1,2,...,m, at which there exists a well defined normal direction, and the IHR(5c) at m' interior points $\underset{\sim}{r}_i$, i=m+1, m+2,..., m+m' leads to the matrix system

$$Ap = Bv + p_{inc} \tag{10}$$

where,

$$A_{ij} = \frac{1}{2}\phi_j(\underset{\sim}{r}_i) - \int_{S^*} \phi_j(\underset{\sim}{r}) \frac{\partial G}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}_i)\, dS_r \tag{11}$$

$$B_{ij} = i\omega\rho_f \int_{S^*} \chi_j(\underset{\sim}{r})G(\underset{\sim}{r},\underset{\sim}{r}_i)\, ds_r \tag{12}$$

for $i=1,2,\ldots, m+m'$; $j=1,2,\ldots, m$,

and $p = \{p_1\ p_2 \ldots p_m\}^T$, $v = \{v_1\ v_2 \ldots v_m\}^T$, and

$$p_{inc} = \{p_{inc}(\underset{\sim}{r}_1)\ \ p_{inc}(\underset{\sim}{r}_2) \ldots p_{inc}(\underset{\sim}{r}_{m+m'})\}^T.$$

Details of a particular choice of basis functions and a method of generating the approximate surface S* are discussed later.

21. Note that the resulting matrices are full, in contrast with the sparse banded matrices that are typical of the numerical solution of differential equation problems by finite element or finite difference methods, and the coefficients, which are frequency dependent, must in general be derived via time-consuming numerical quadrature. Also, since collocation has been employed to obtain sufficient relations between the parameters, the system is not symmetric.

22. The system of equations (10) is sufficient to determine completely the acoustic field in two particular cases of interest; namely for radiation ($p_{inc}(\underset{\sim}{r}) \equiv 0$) where the surface velocity is specified, and for scattering where the acoustic impedance of the surface is specified, most usually assumed to be a perfectly hard or rigid surface ($v(\underset{\sim}{r}) \equiv 0$).

23. However, in general the vibrational response of the structure must be taken into account, in which case equation (10) merely provides a relationship between acoustic pressure and normal velocity at the structure surface.

STRUCTURAL EQUATIONS

24. The region enclosed by the surface S is assumed to contain an elastic structure whose motion is governed by the linear equations of elasticity,

$$\text{div}\ \underset{\sim}{\sigma}(\underset{\sim}{r},t) + \underset{\sim}{F}(\underset{\sim}{r},t) = \rho_s \frac{\partial^2}{\partial t^2} \underset{\sim}{U}(\underset{\sim}{r},t) \tag{13}$$

where $\underset{\sim}{\sigma}(\underset{\sim}{r},t)$ is the stress tensor, $\underset{\sim}{F}(\underset{\sim}{r},t)$ represents external forces, ρ_s is the structure density, and $\underset{\sim}{U}(\underset{\sim}{r},t)$ is the particle displacement.

25. The discretisation of these equations by the finite element method is now a familiar technique[13] and will not be described in detail here. An approximation to the structural displacement is assumed in the form

$$\underset{\sim}{U}(\underset{\sim}{r},t) = e^{i\omega t} \sum_{i=1}^{n} \underset{\sim}{u}_i\ \Phi_i(\underset{\sim}{r}) \tag{14}$$

where each $\Phi_i(\underset{\sim}{r})$ is a diagonal matrix (of order d, the spatial dimension of the problem) of basis functions defined throughout the volume of the structure. These basis functions are normally chosen such that they vanish at all but one node of an element subdivision of the structure, in which case the vector parameter $\underset{\sim}{u}_i$ describes the displacement at that node. If each component of the parameter $\underset{\sim}{u}_i$ has the same associated basis function, as is frequently the case in practice, the matrix $\Phi_i(\underset{\sim}{r})$ may be replaced by a scalar basis function $\phi_i(\underset{\sim}{r})$.

26. The finite element equations for the structure with the harmonic time dependence omitted, are then of the form

$$(K + i\omega C - \omega^2 M)\; q = f^{(k)} + f^{(p)} \tag{15}$$

where K, C and M are stiffness, damping and mass matrices respectively, q is the vector of parameters $\underset{\sim}{u}_i$, $f^{(k)}$ is a consistent load vector derived from known applied forces, and $f^{(p)}$ is a consistent load vector representing the acoustic fluid pressure acting on the fluid-structure boundary. Explicitly, the vector $f^{(p)}$ has components

$$f_i^{(p)} = - \int_S p(\underset{\sim}{r})\; \Phi_i(\underset{\sim}{r})\; \underset{\sim}{n}\; dS_r \tag{16}$$

where this notation is used to define simultaneously all the components of $f^{(p)}$ associated with the vector parameter $\underset{\sim}{u}_i$.

27. The matrices K and M and usually C are symmetric and banded and if structural damping is ignored, the system is real. If there is no acoustic loading on the structure or if the structure is only subjected to a static fluid pressure, the system of equations (15) alone defines the displacement of the structure in terms of the applied loads. However, in the case of dynamic fluid-solid interaction, these equations only relate structural displacements to the acoustic pressure at the fluid-structure interface.

COUPLED EQUATIONS OF MOTION

28. The complete solution of the fluid-structure interaction problem may now be described by combining the acoustic equations (10) with the structural equations (15). Substitution of the representation (8) for the surface acoustic pressure into (16) leads to

$$f^{(p)} = -Lp \tag{17}$$

where

$$L_{ij} = \int_S \psi_j(\underset{\sim}{r})\; \Phi_i(\underset{\sim}{r})\; \underset{\sim}{n}\; dS_r \tag{18}$$

for i=1,2, ..., n; j=1,2, ..., m, which ensures continuity of sound pressure between the structural and acoustic models.

29. Continuity of normal surface velocity is achieved by matching at some set of surface nodes, $\underset{\sim}{r}_i'$, i=1,2, ..., k, the approximations (9) and (14) to give

$$Xv = i\omega L'q \tag{19}$$

where

$$X_{ij} = \chi_j(\underset{\sim}{r}'_i) \tag{20}$$

for i=1, 2, ..., k; j=1,2, ..., m, and

$$L'_{ij} = \Phi_j(\underset{\sim}{r}'_i)\, \underset{\sim}{n}'_i \tag{21}$$

for i=1,2, ..., k; j=1,2, ..., n, with $\underset{\sim}{n}'_i$ being the outward normal direction at $\underset{\sim}{r}'_i$.

30. Normally the basis functions $\psi_i(\underset{\sim}{r})$ and $\chi_i(\underset{\sim}{r})$ would be chosen to be the same and to interpolate m surface points. These would then be the natural collocation points for the SHE and would also be the positions at which to ensure the continuity of velocity. In this case X would be the identity matrix. If these acoustic nodes coincide with some or all of the surface structural nodes then the non-zero elements of the matrix L' are simply components of the outward normal directions at the acoustic nodes.

31. The solution of equations (10), (15), (17) and (19) may be accomplished in a number of ways. Elimination of the structural displacement vector leads to a combined matrix equation in acoustic form

$$(A + DL)\, p = Df^{(k)} + p_{inc} \tag{22}$$

where

$$D = i\omega BX^{-1}L'\,(K + i\omega C - \omega^2 M)^{-1}. \tag{23}$$

Once the surface pressure has been determined the structural motion follows directly from

$$q = (K + i\omega C - \omega^2 M)^{-1}\,(f^{(k)} - Lp). \tag{24}$$

Initial elimination of the acoustic pressure leads to a perturbation of the structural equations,

$$(K + i\omega C - \omega^2 M + i\omega LA^{-1}BX^{-1}L')\, q = f^{(k)} - LA^{-1}\, p_{inc} \tag{25}$$

with the surface pressure then being given by

$$p = A^{-1}\,(\omega BX^{-1}L'\, q + p_{inc}).$$

32. If it is necessary to form non-square matrices A and B to overcome non-uniqueness problems, the inverse of matrix A is to be understood in a generalised least squares sense.

Let

$$LA^{-1}BX^{-1}L' = \omega(R_1 + iR_2) \tag{27}$$

then equation (25) becomes

$$(K + i\omega(C + \omega R_1) - \omega^2 (M + R_2))\, q = f^{(k)} - LA^{-1}\, p_{inc} \qquad (28)$$

and the effect of the fluid can be seen as added mass and damping terms. If the fluid is considered incompressible (the low frequency limit, $k = 0$) and hence unable to sustain a sound field, the matrices A and B become totally real and imaginary respectively and the added damping term is, as expected, identically zero. In this case it is the Laplace equation which governs the fluid behaviour, for which there are no non-uniqueness or non-existence problems associated with an integral equation formulation.

33. From a computational point of view the system of equations (22) is dense, complex, unsymmetric and of dimension $(m+m') \times m$. As an intermediate step it is necessary to form the products DL and $Df^{(k)}$, not by computing explicitly the inverse matrix $(K + i\omega C - \omega^2 M)^{-1}$ but by solving an $nd \times nd$ system of linear equations with $(K + i\omega C - \omega^2 M)$ as the coefficient matrix and the columns of L and also $f^{(k)}$ as the right hand side vectors. Efficient routines are available, due to the symmetry and bandedness of the coefficient matrix, to carry out this procedure the result of which is retained for subsequent evaluation of the structural displacement parameters.

34. Now consider the system of equations (25). Due to the acoustic perturbing term, the system no longer possesses the structural features of bandedness and symmetry, but is relatively dense, complex, unsymmetric and of dimension $nd \times nd$. As an intermediate step in this calculation it is necessary to form $A^{-1}B$ and $A^{-1}p_{inc}$ by solving an $(m + m') \times m$ dense complex system with multiple right hand sides using a least squares procedure.

35. Since in general $n \gg (m + m')$ it would appear computationally more efficient to formulate the coupled solution as the system of equations (22) and this is preferred here, although some authors have adopted the alternative approach.[8,12]

36. For many real complex structures submerged in water, particularly when no symmetry is present, the finite element model would necessarily have a large number of degrees of freedom and thus excessive computer cost may restrict either of the above approaches. In this case a modal approach may be used to advantage,[37] whereby the structure displacement is approximated by a linear combination of the dominant in-vacuo, undamped normal modes of the structure. The modal frequencies (λ_i) and mode shapes (θ_i) are efficiently obtained from equation (15) with the load and damping terms set to zero,

$$(K - \lambda_i^{\,2} M)\, \theta_i = 0. \qquad (29)$$

A convenient normalisation is usually

$$\theta_i^{\,T} M\, \theta_i = 1 \qquad (30)$$

$$\theta_i^{\,T} K\, \theta_i = \lambda_i^{\,2} \qquad (31)$$

with the orthogonality relations

$$\theta_i^T M \theta_j = \theta_i^T K \theta_j = \theta_i^T \theta_j = 0 \qquad (32)$$

holding for $i \neq j$.

If the structure displacement is now expressed in terms of N of these modes as

$$q = \Theta \alpha \qquad (33)$$

where α is a vector of parameters and Θ is the matrix whose columns are the N normal modes, the acoustic and structural forms of the coupled problem become respectively,

$$(A + SL)\, p = Sf^{(k)} + p_{inc} \qquad (34)$$

where

$$S = i\omega BX^{-1}L'\, \Theta\, (\Lambda + i\omega\, \Theta^T C\, \Theta - \omega^2 I)^{-1}\, \Theta^T \qquad (35)$$

and

$$(\Lambda + i\omega\, \Theta^T (C + LA^{-1}BL')\, \Theta - \omega^2 I)\, \alpha = \Theta^T (f^{(k)} - LA^{-1} p_{inc}). \qquad (36)$$

Λ is the diagonal matrix with elements λ_i^2.

37. These systems are similar to those given previously except that if only a small number of dominant eigenmodes are chosen to represent the displacement, the matrices derived from the structural model are of smaller size ($N \ll n$). Of course the eigensolutions of the structure also have to be determined, but this need only be carried out once for a structure while a coupled solution might be required at a number of frequencies. If no internal damping is present or if present can be represented in a particularly simple manner, the matrix inversion required in equation (35) becomes trivial.

38. Once the surface pressure and structural displacement (and hence normal surface velocity) have been evaluated, the pressure at any position in the fluid may be determined directly from the Helmholtz equation (5a) via numerical quadrature in an analogous manner to the derivation of the coefficients of the acoustic matrices A and B. For positions at large distances, R, in terms of the acoustic wavelength, from the structure the radiated or scattered part of the acoustic field has an angular distribution independent of R. The dependence on R is of the form $\exp(-ikR)/R$ which may be factored out of the Helmholtz integral to simplify its evaluation.[38]

39. Thus equations which describe a complete solution of the coupled fluid-structure interaction problem have been presented for a structure either subjected to internal applied forces or acting as a scatterer of an incident acoustic wave. Normally one of the vectors $f^{(k)}$ and p_{inc} will be identically zero although the formulation does not require this to be so.

NUMERICAL IMPLEMENTATION AND RESULTS

40. In order to evaluate the coefficients of the matrices A and B in the acoustic equations (10) it is necessary to define a surface S^*, an approximation to S, choose the basis functions $\phi_i(\underset{\sim}{r})$ and $\chi_i(\underset{\sim}{r})$ appearing in the surface pressure and normal surface velocity representations respectively, choose a set of collocation points, and finally select an efficient numerical integration routine.

41. A surface S^* may be defined as the continuous union of m sub-regions S_i^*, each of which interpolates to some degree a set of nodes lying on S. If the transformation

$$\underset{\sim}{r} = \underset{\sim}{r}(u,v) \tag{37}$$

maps a point (u,v) in a local surface coordinate system for the subregion S_i^* to a position $\underset{\sim}{r} = (x, y, z)$ in the global cartesian coordinate system for the surface S, then an integral of the form

$$I = \int_S f(\underset{\sim}{r})\, dS_r \tag{38}$$

may be approximated by

$$I \approx \sum_{i=1}^{m} \int_{S_i^*} f(\underset{\sim}{r}) \sqrt{D}\, du dv \tag{39}$$

where

$$D = D_1^{\,2} + D_2^{\,2} + D_3^{\,2} \tag{40}$$

and

$$D_1 = \frac{\partial(y, z)}{\partial(u, v)}, \quad D_2 = \frac{\partial(z, x)}{\partial(u, v)}, \quad D_3 = \frac{\partial(x, y)}{\partial(u, v)}. \tag{41}$$

$(D_1, D_2, D_3)/\sqrt{D}$ is the unit normal to S_i^* at (u, v), which is required to evaluate the normal derivative of the Green's function.

42. In particular, triangular subregions of S are interpolated quadratically through,

$$\begin{aligned} \underset{\sim}{r} = {} & (1-u-v)(1-2u-2v)\, \underset{\sim}{r}_1 + u(2u-1)\, \underset{\sim}{r}_2 + v(2V-1)\, \underset{\sim}{r}_3 \\ & + 4uv\underset{\sim}{r}_4 + 4v(1-u-v)\underset{\sim}{r}_5 + 4u(1-u-v)\underset{\sim}{r}_6 \end{aligned} \tag{42}$$

where the nodes $\underset{\sim}{r}_1$, $\underset{\sim}{r}_2$ and $\underset{\sim}{r}_3$ map to the vertices (0, 0), (1, 0) and (0, 1) respectively of a triangle in the (u, v) surface coordinate system, and the nodes $\underset{\sim}{r}_4$, $\underset{\sim}{r}_5$ and $\underset{\sim}{r}_6$ map to the midside positions $(\frac{1}{2}, \frac{1}{2})$ $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ respectively. To ensure an invertible mapping with the Jacobian being non-vanishing within the triangle certain restrictions are necessary upon the positions of the midside nodes $\underset{\sim}{r}_4$, $\underset{\sim}{r}_5$ and $\underset{\sim}{r}_6$.[39] A similar transformation will map biquadratically nine reasonably positioned nodes to the vertices, midside positions and centre of a unit square in the local surface coordinate system.

43. In the present implementation of the CHIEF method each subregion S_i^* is thus defined by quadratically interpolating either six or nine nodes lying in S to give triangular or quadrilateral subregions respectively as appropriate. Piecewise constant approximations are taken for the surface pressure and normal surface velocity, such that the basis functions are defined as

$$\psi_i(\underset{\sim}{r}) = \chi_i(\underset{\sim}{r}) = \begin{cases} 1 & \underset{\sim}{r} \text{ in } S_i^* \\ 0 & \text{otherwise} \end{cases} \tag{43}$$

44. In the standard finite element terminology, these elements are of the superparametric type with the geometry being specified to a greater accuracy than the unknown functions.

45. The collocation points are naturally taken as the centroids of the surface elements and the piecewise constant computed solution is usually observed to best approximate the true solution at these positions. Although such a function approximation may beem rather crude, it has successfully formed the basis of many practical acoustic radiation/scattering computer programs.[14-17]

46. Numerical experience with a spherical surface indicates that quadratic surface elements lead to an increase in accuracy over a piecewise planar surface by a factor of at least 20, whilst a linear function approximation gives little or no improvement over piecewise constant basis functions. The latter behaviour is analogous, for integral equations, to the midpoint rule being more accurate than the trapezium rule for numerical integration. The current interest in integral equation methods will no doubt provide more insight into the gains to be made with higher order approximations. In particular, Burton[31] has suggested the use of isoparametric bicubic spline elements as the basis of an improved acoustic program.

47. The coefficients of the acoustic matrices must be evaluated through numerical quadrature. Singular integrands arise whenever the collocation point $\underset{\sim}{r}_i$ lies within the region of integration S_j^*, that is, for the diagonal matrix elements A_{ii} and B_{ii}, i=1,2, ..., m derived from the SHE.

48. The dominant part of the singularity in the diagonal elements of the A matrix may be removed by rewriting the double layer potential term as

$$\int_S p(\underset{\sim}{r}) \frac{\partial G}{\partial n_r} (\underset{\sim}{r},\underset{\sim}{r}') \, dS_r = \int_S \left\{ p(\underset{\sim}{r}) \frac{\partial G}{\partial n_r} (\underset{\sim}{r},\underset{\sim}{r}') - p(\underset{\sim}{r}') \frac{\partial G_o}{\partial n_r} (\underset{\sim}{r},\underset{\sim}{r}') \right\} dS_r - \frac{1}{2} p(\underset{\sim}{r}') \tag{44}$$

where

$$G_o(\underset{\sim}{r},\underset{\sim}{r}') = \frac{1}{4\pi|\underset{\sim}{r}-\underset{\sim}{r}'|} \tag{45}$$

is the free-space Green's function for the Laplacian operator, and use has been made of the identity,

$$\int_S \frac{\partial G_o}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}')\, dS_r = -\frac{1}{2} \qquad \underset{\sim}{r}' \text{ on } S. \tag{46}$$

The weak singularity of $\frac{\partial G}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}')$ at $\underset{\sim}{r}=\underset{\sim}{r}'$ has been subtracted out by one of the same form of $\frac{\partial G_o}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}')$.

49. Then, for the particular basis functions defined by (43),

$$A_{ii} = 1.0 + \sum_{j=1}^{m} \int_{S_j^*} \frac{\partial G_o}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}_i)\, dS_r - \int_{S_i^*} \frac{\partial G}{\partial n_r}(\underset{\sim}{r},\underset{\sim}{r}_i)\, dS_r. \tag{47}$$

50. Numerically, the same integration rule should be employed for both integrals over the subregion S_i^* to achieve subtaction of the singularity. Unfortunately, it is not possible to treat the weak singularity of the single layer potential term in a similar manner, and a simpler but less reliable scheme is employed. The region S_i^* is further subdivided into subregions such that the *collocation point $\underset{\sim}{r}_i$ lies at a vertex of each subregion. In the surface co*ordinate system (u, v), the collocation point either lies at the position $(\frac{1}{2}, \frac{1}{2})$ in the unit square or at $(\frac{1}{3}, \frac{1}{3})$ within the standard triangle. The unit square is divided into four subsquares by the lines $u = \frac{1}{2}$ and $v = \frac{1}{2}$ and the standard triangle is divided into three subtrianges with the point $(\frac{1}{3}, \frac{1}{3})$ being a common vertex. The integration rules adopted are Gaussian rules with degree of precision 3 for non-singular integrands and degree of precision 7 within each subregion for singular integrands.[40,41] Higher order rules were found to give no significant increase in accuracy whilst with lower order rules the final solution deteriorated in accuracy.

51. Advantage may be taken of any geometrical symmetry in the body when the surface pressure and normal surface velocity have an identical symmetry. The same unknown parameters may be associated with a number of surface elements S_i^* and consequently it is only necessary to collocate at the centroid of one of those elements. In particular for complete axial symmetry, as in the examples discussed below, this technique may be and is employed. In such cases however, it would be considerably more efficient to take full advantage of the symmetry and reformulate the problem in terms of a line rather than a surface integral equation.[16]

52. As an illustration of the numerical method, consider a plane acoustic wave incident upon a submerged homogeneous isotropic spherical shell with no material damping enclosing a vacuum. Two particular cases are presented; firstly, a relatively thick shell which acts very much like a rigid scatterer, and secondly a thin shell where the structure has a more pronounced effect on the acoustic field.

53. Axisymmetric finite elements are used to model the shell for which the mass and stiffness matrices have been defined many times.[13] A six-noded isoparametric quadrilateral with quadratic variation in the circumferential direction and linear variation in the radial direction was found to give acceptable results with one element through the thickness of the shell. The acoustic and structural elements are illustrated in Figure 1. Structural nodal lines on the outer spherical surface coincide with acoustic element boundaries in order to facilitate computation of the interaction matrix L.

54. Firstly, consider a spherical steel shell of outer radius 2.0 cm and inner radius 1.5 cm submerged in water. The frequency of the incident plane wave is chosen such that the acoustic wavenumber k ($= \omega/c$) is unity. The scattered surface pressure is initially computed assuming the outer surface of the shell to be perfectly rigid, in which case equations (10) alone fully describe the acoustic field (with $v \equiv 0$). In Figure 2 the continuous lines are the real and imaginary parts of the exact solution,[42] while the computed solution is shown using 18 elements in the direction of θ and 24 elements in each band around the axis of symmetry.

55. When the response of the elastic structure is taken into account, modelled with 18 structural elements, the results are as illustrated in Figures 3 and 4. The exact solution, shown as the continuous lines, is obtained through a separation of variables analysis following Goodman and Stern.[1] In Figure 3 the exact solution for the rigid surface is also shown for comparison.

56. Now consider a thin spherical shell of outer radius 5.0 cm and inner radius 4.921875 cm. The scattered surface pressure when the outer surface is considered to be rigid is given in Figure 5, using the same number of acoustic elements as for the smaller sphere. Figures 6 and 7 show the effect of including the vibrational response of the steel shell through 18 axisymmetric structural elements. The exact rigid sphere solution is also superimposed upon Figure 6 to emphasis the stronger fluid-solid interaction in this case. This is also evident from a comparison of the magnitude of the surface displacements in the two cases.

57. In neither of these problems is it necessary to consider interference from interior eigenfunctions. For a spherical fluid-solid interface non-uniqueness can only occur when the non-dimensional parameter ka (wavenumber x radius of sphere) is a zero of the n-th order spherical Bessel function of the first kind, j_n, for some value of n.

CONCLUSIONS

58. By combining equations derived from a finite element analysis of a vibrating elastic structure with those from an integral equation representation of an infinite exterior acoustic field, an analysis of the coupled dynamic interaction problem has been shown to be feasible.

59. The structural and acoustic models may be chosen completely independently although the procedure does simplify if interpolation nodes are made to coincide, the coupling being defined through three interaction matrices X, L and L'. Thus advantage may be taken of existing computer program packages designed to solve the two uncoupled problems. In particular, it is hoped to combine the PAFEC[43] structural analysis program with the acoustic radiation/scattering program described in the previous section.[38]

60. Some test examples for which analytic solutions are available have demonstrated the practicability and accuracy of the approach. It is hoped in the near future to be able to compare numerical and experimental results for some more complex structures in water. If an unacceptably large numer of degrees of freedom for the structural model are required, it may then be necessary to consider the alternative modal approach.

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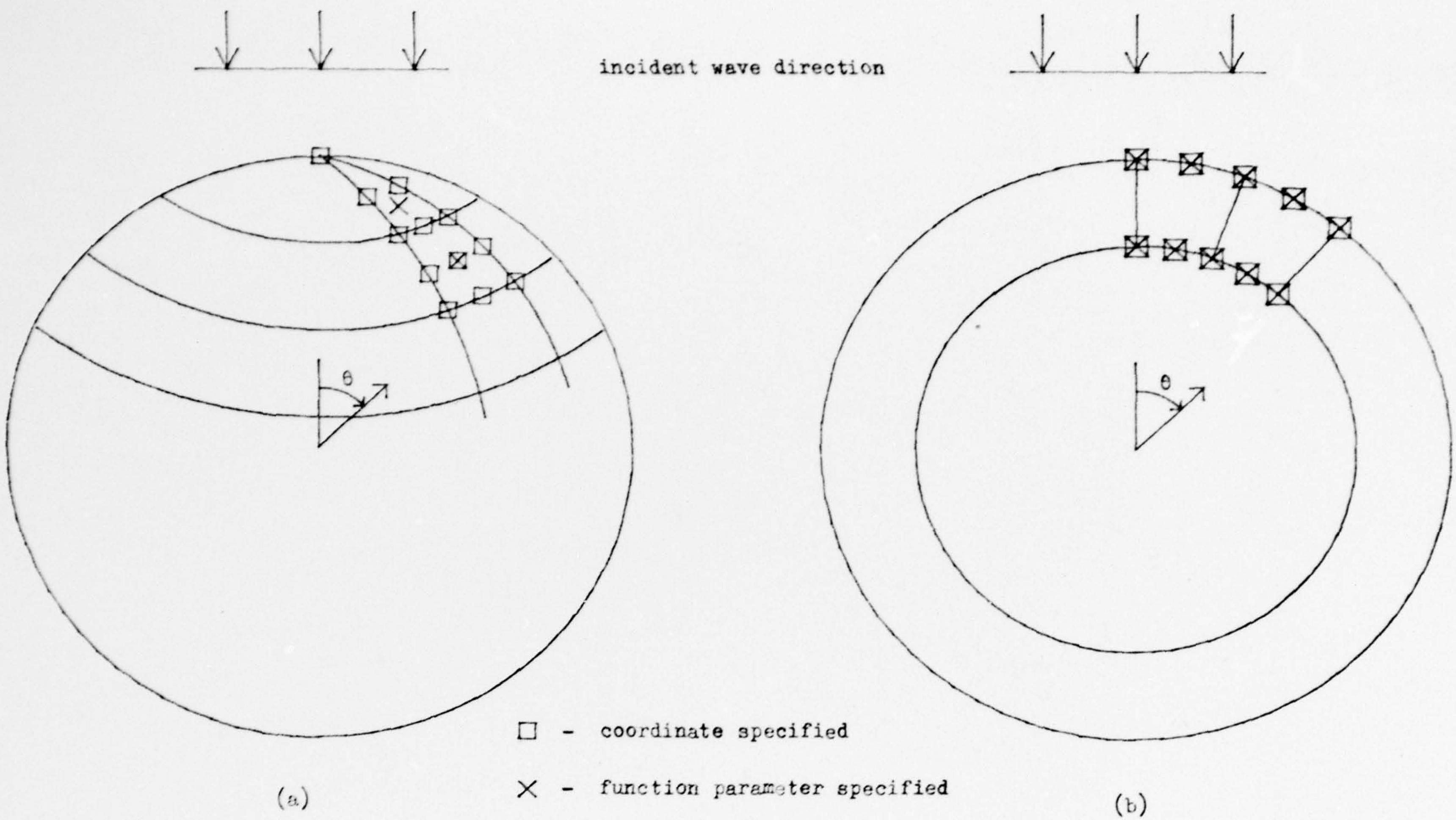

FIGURE 1 (a) Surface acoustic finite elements; (b) Structural finite elements

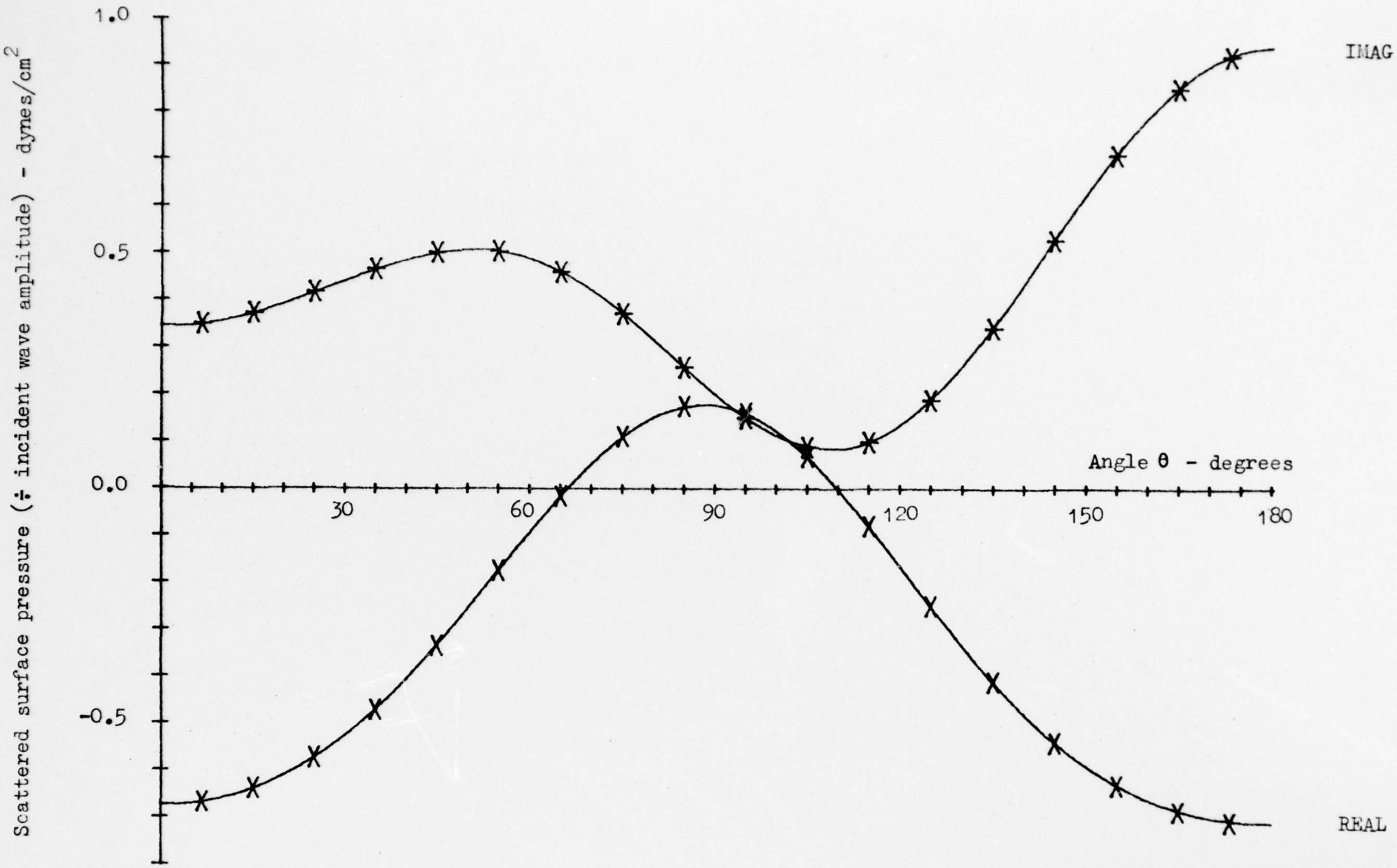

FIGURE 2 Scattered surface pressure due to a plane wave incident upon a rigid sphere; wavenumber x radius = 2.0. Solid lines - analytical solution; crosses - computed solution.

FIG. 2.

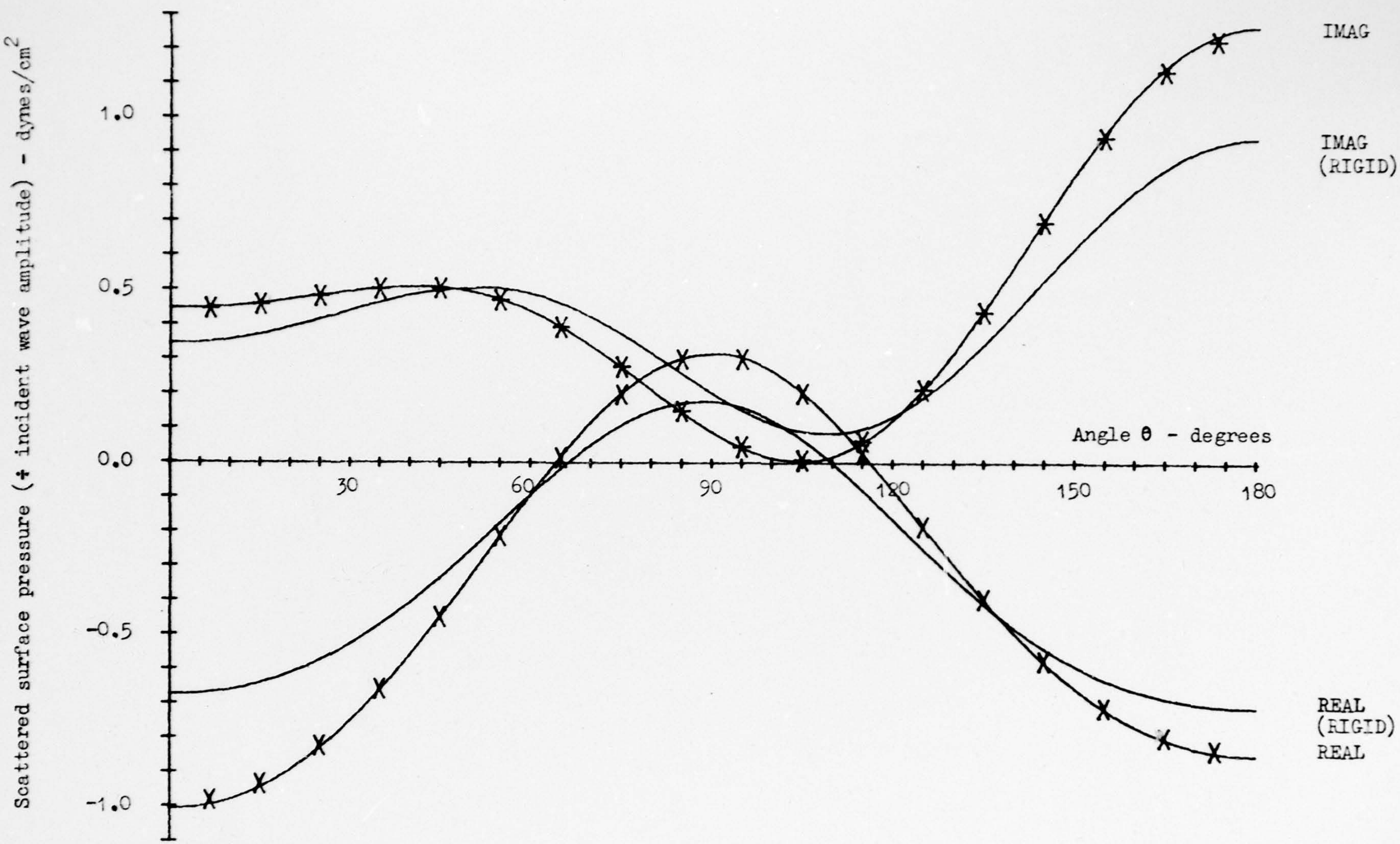

FIGURE 3 Scattered surface pressure due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 2.0 cm, inner radius = 1.5 cm. Solid lines - analytical solution; crosses - computed solution.

FIG: 3.

FIG. 4

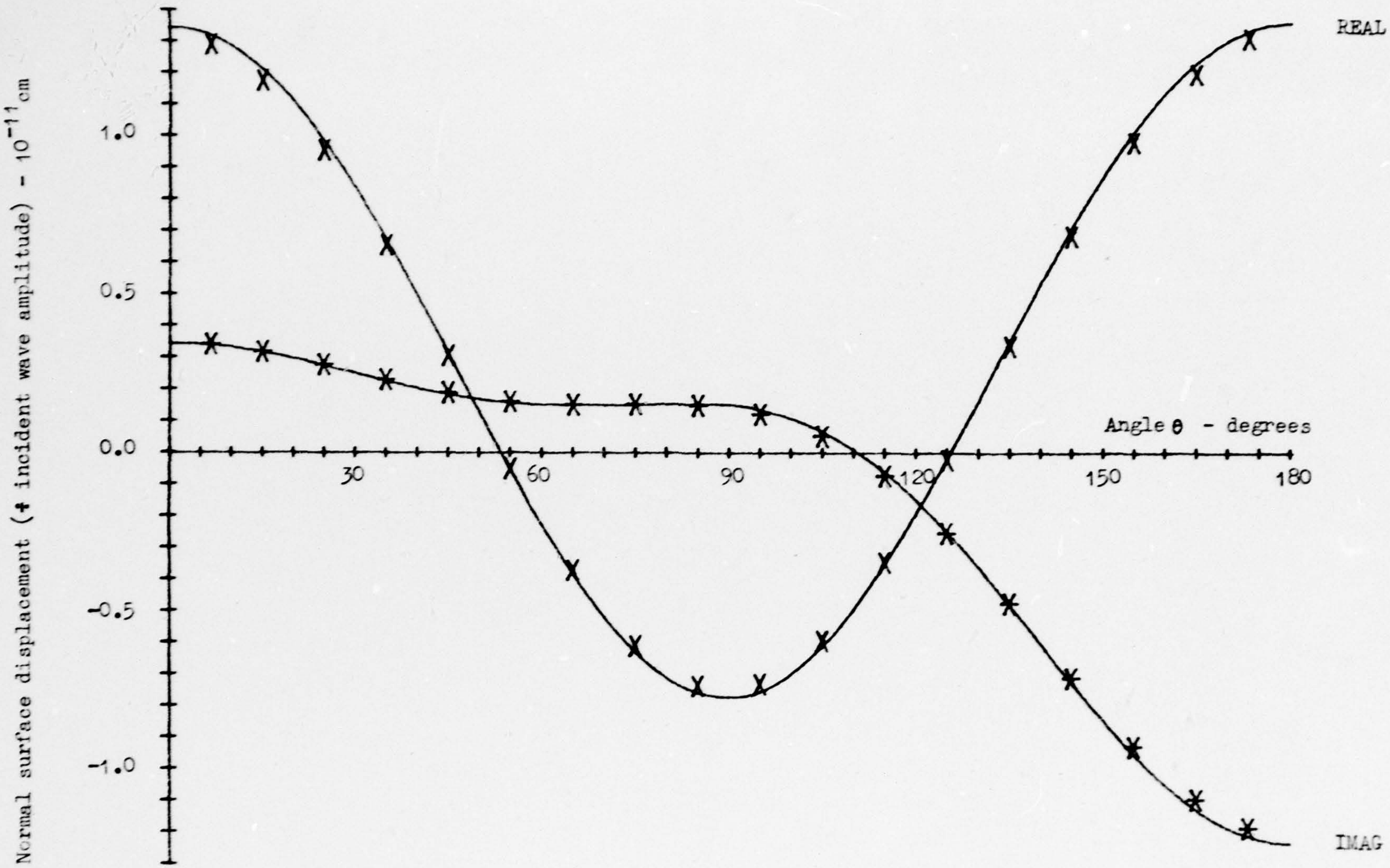

FIGURE 4 Normal surface displacement due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 2.0 cm, inner radius = 1.5 cm. Solid lines - analytical solution; crosses - computed solution.

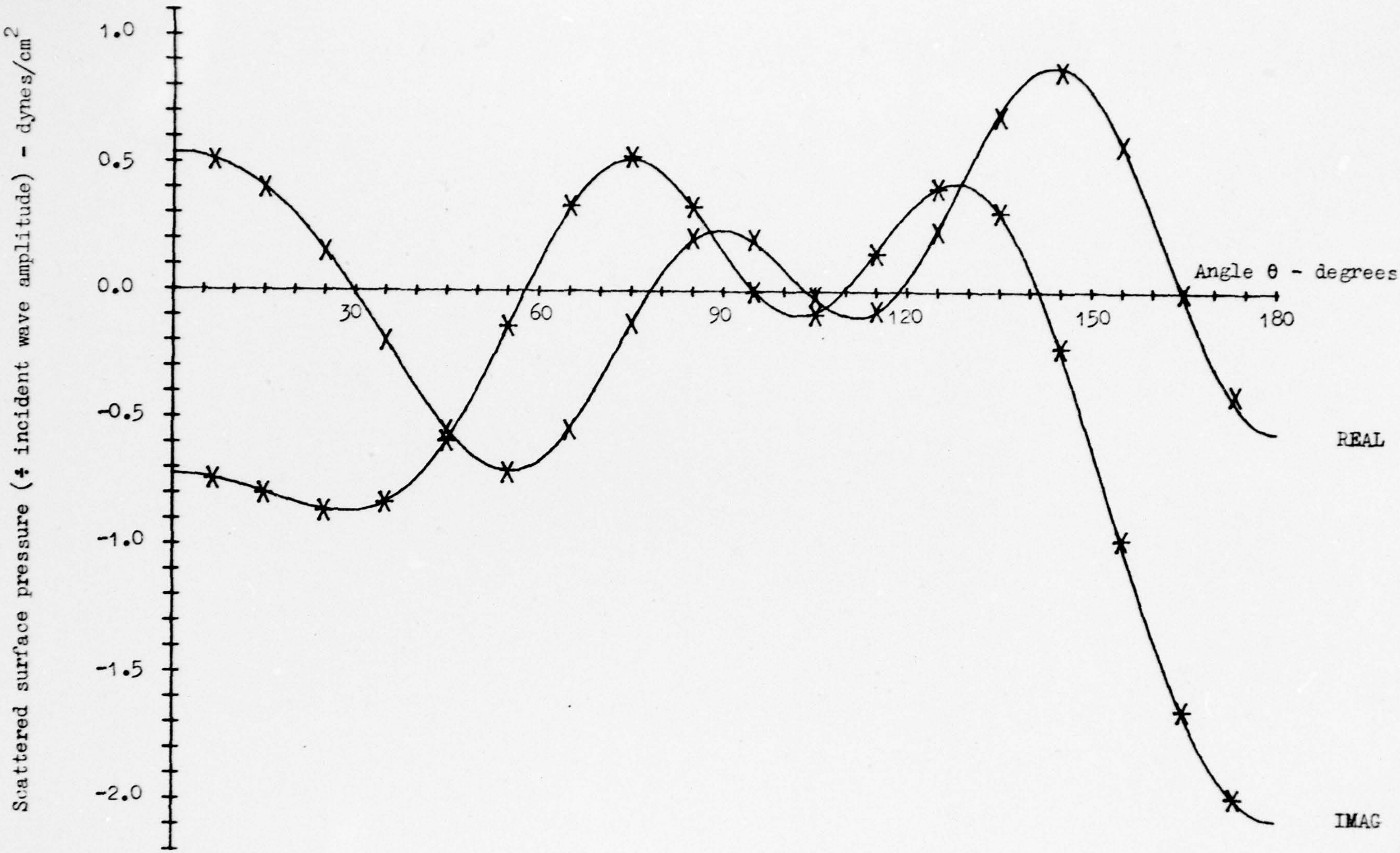

FIGURE 5 Scattered surface pressure due to a plane wave incident upon a rigid sphere; wavenumber x radius = 5.0. Solid lines - analytical solution; crosses - computed solution.

FIG. 5.

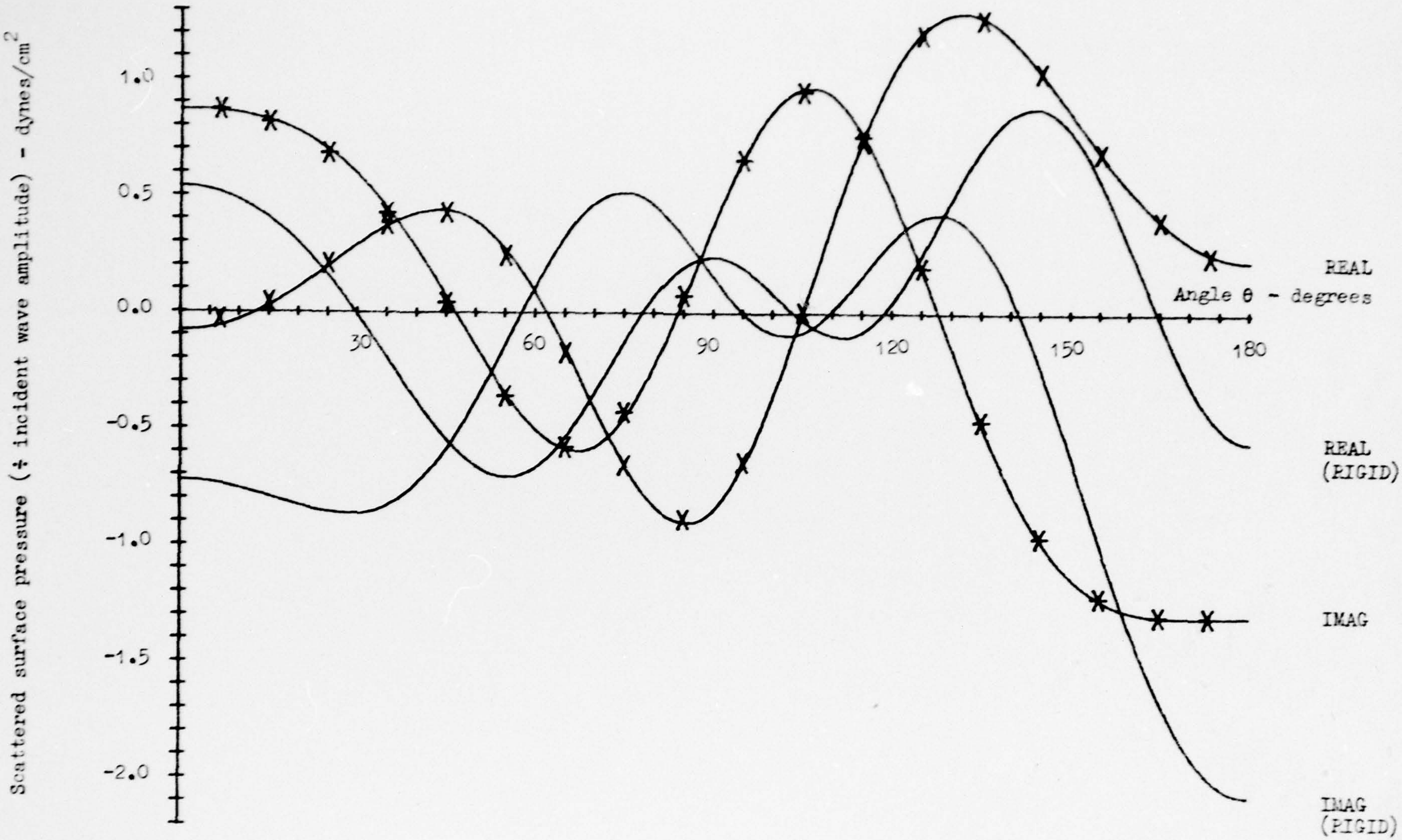

FIGURE 6 Scattered surface pressure due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 5.0 cm, inner radius = 4.921875 cm. Solid lines - analytical solution; crosses - computed solution.

FIG. 6.

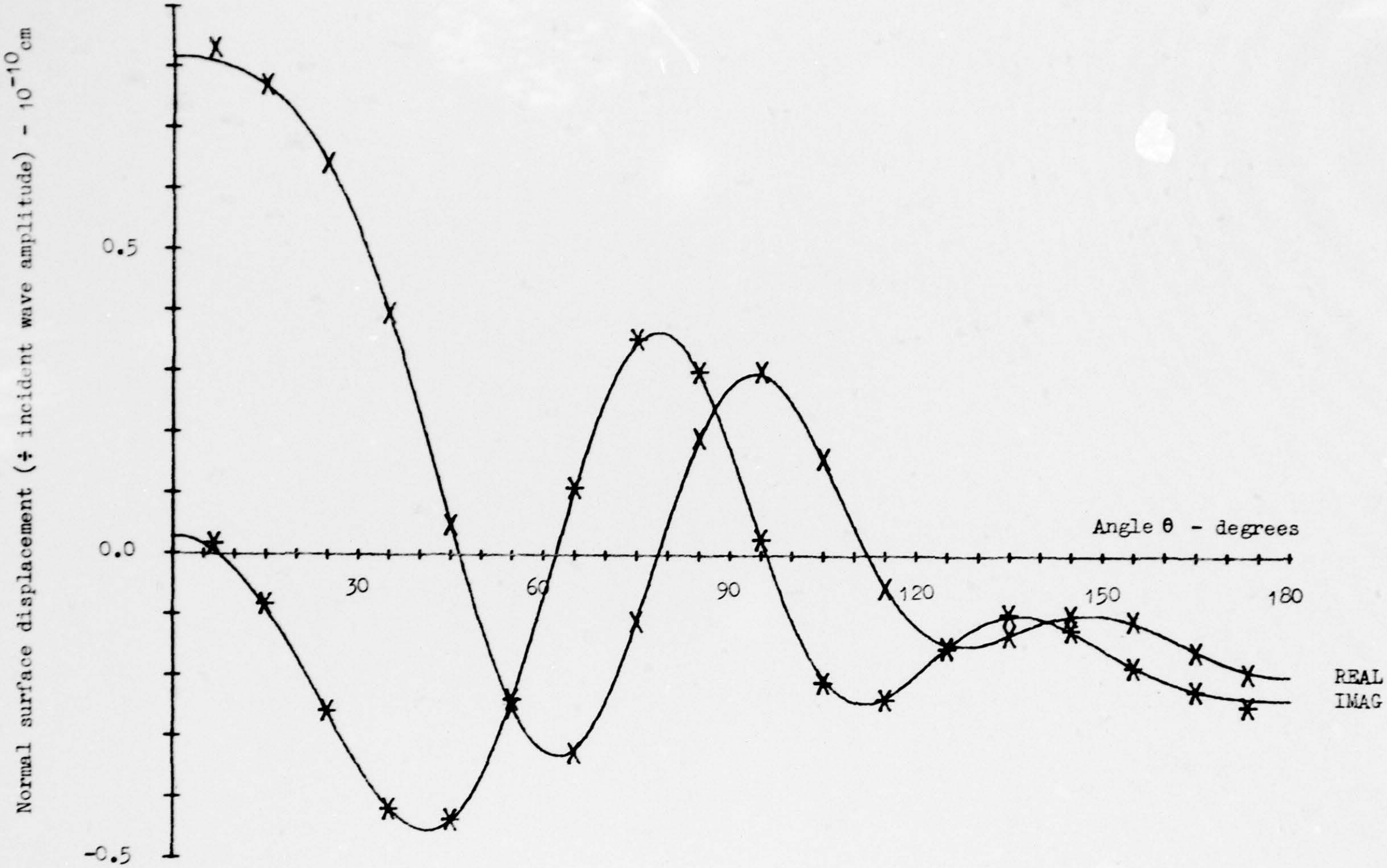

FIGURE 7 Normal surface displacement due to a plane wave incident upon an elastic spherical shell; wavenumber = 1.0, outer radius = 5.0 cm, inner radius = 4.921875 cm. Solid lines - analytical solution; crosses - computed solution.

FIG. 7.

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Abstract This report presents a numerical technique for the linear dynamic analysis of a finite elastic structure immersed in an infinite homogeneous acoustic medium. A finite element analysis of the structure is matched at the structure-fluid interface with an integral equation representation of the exterior acoustic field, leading to a coupled system of equations which may be cast in either acoustic or structural form. The former approach is preferred here for which numerical results are presented when the method is applied to plane wave scattering by thick and thin elastic spherical shells.

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